

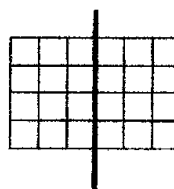
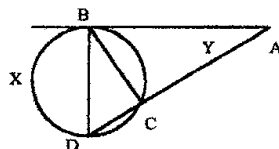
**2007 ARML RUNOFF**  
**Round 1**  
**(45 Minutes)**  
**NO CALCULATORS**

Print Name \_\_\_\_\_  
 School \_\_\_\_\_ Grade \_\_\_\_\_  
 Home Phone \_\_\_\_\_  
 e-mail \_\_\_\_\_

**Answers**

1. Compute  $(x + y)$  if  

$$\frac{x^2}{2007} - \frac{y^2}{2007} = 1 \text{ and } \frac{x}{2007} - \frac{y}{2007} = \frac{1}{2}$$
2. Points  $(3,2)$  and  $(7,2)$  are two vertices of a triangle. Find the locus (equation) of the third vertex if the area of the triangle is 12 square units.
3. Solve for  $x$ :  $8^{x+2} = 8^x + 252$
4. A positive number is represented by a two-digit decimal numeral. If 6 times the sum of the digits is added to the number, the digits are in reverse order. What is the number?
5. Let  $f$  be a polynomial function such that for all  $x$ ,  $f(x^2 - 1) = x^4 - 5x^2 + 3$ . For all real  $x$ , find  $f(x^2 + 1)$ .
6. Solve for  $x$ :  $\log_2(5x - 4) - \log_2(x^2 - 1) = 1$
7.  $(2 + 2i)$  is a root of the equation  $x^4 - 4x^3 - 6x^2 + 8x = 16$ . Find the other three roots.
8. Find the measure of  $\widehat{X}$ .  
 Find the  $m\angle Y$ .  
  
 Given:  $m\angle BDC = 42^\circ$   
 $m\angle CBD = 44^\circ$
9. Find  $\theta$ :  $\sin 2\theta \cos 18^\circ + \cos 2\theta \sin 18^\circ = \frac{\sqrt{3}}{2}$
10. ABCD is a quadrilateral with  $m\angle DAB = 135^\circ$ ,  $BC = 3$ , and  $CD = 2\sqrt{2}$ . Compute the maximum possible area of ABCD.
11. A square is divided into 24 congruent rectangles as shown. On each side of the dotted line 4 rectangles are chosen at random and colored black. The square is then folded over the dotted line. Compute the probability that exactly one pair of black rectangles is coincident.
12. Solve for  $x$ :  $(x - 3) < x^2(x - 3)$ .



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**2007 ARML RUNOFF**  
 Round 2  
 (45 Minutes)  
**NO CALCULATORS**

Print Name \_\_\_\_\_  
 School \_\_\_\_\_ Grade \_\_\_\_\_  
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 e-mail \_\_\_\_\_

**ANSWERS**

13. How many integers between 20,000 and 30,000 are perfect squares?

14. Solve the following system for  $(x, y)$

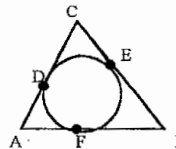
$$\begin{cases} y = 7x + 6 \\ y = x^3 \end{cases}$$

15. Solve for  $x$ :

$$x + \log_3 9 \cdot \log_5 4 \cdot \log_9 5 = \log_3 36$$

16. Find  $BC$ . (not drawn to scale.)

Given:  $CD = 6$ ;  $DA = 8$ ;  $AB = 18$



17. Let  $P$  be the parabola with vertex at the origin and directrix  $y = -1$ .

Compute the number of lattice points on  $P$  whose distance from  $(0, 1)$  is less than or equal to 122.

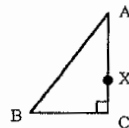
18. Solve for  $\theta$ :

$$\frac{\sin \theta}{1 + \cos \theta} = 1$$

19. In right triangle  $ABC$ ,  $AC = 8$  and  $BC = 6$ .

Point  $X$  is on  $AC$  equidistant from  $A$  and  $B$ .

Find  $CX$ .



20. A large cube is dipped into red paint and then divided into 125 smaller congruent cubes. One of the smaller cubes is then randomly selected. What is the probability that the selected cube will have at least 25% of its surface area painted red?

21. Solve for  $x$ :  $x(x + 214) = 2007$ .

22. Give the 3 cube roots of  $-27$ .

23. The arithmetic mean of nine numbers is 54. If two numbers,  $A$  and  $B$  are added to the list, the mean of the eleven number list becomes 66. What is the mean of  $A$  and  $B$ ?

24. Factor over the integers:  $x^4 - 7x^2 + 9$

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